The influence of isotropic strengthening (according to Taylor) on the strain diagram of a polycrystal is exhibited in Fig. 3, where curves 1-4 correspond to curves 1, 2, 5, 6 in Fig. 2. The dashed lines correspond to the value of the quantity $b/G = 0.02$ (b is the isotropic strengthening factor, and G is the shear modulus), and the dash-dot line to $b/G = 0.10$.

It follows from the dependences presented that the mean grain dimension exerts the fundamental influence on the behavior of a polycrystalline aggregate consisting of crystallites of different dimensions. The dependence of the strain diagram on the nature of the grain dimension distribution turns out to be negligible.

LITERATURE CITED

1. O. A. Volokhovskaya and V. V. Podalkov, "Microstresses and initial plasticity boundary in a polycrystalline material," Izv. Vyssh. Uchebn. Zaved., Mashinostroenie, No. 10 (1977).
2. O. V. Volokhovskaya, "On the question of the plasticity of polycrystalline materials," Izv. Vyssh. Uchebn. Zaved., Mashinostroenie, No. 11 (1977).
3. O. V. Volokhovskaya and V. V. Podalkov, "On the determination of the macroscopic elastic limit in a statistical model of a polycrystalline material," Abstracts of Reports All-Union Conf. on Optimization and Reliability Problems in Structural Mechanics [in Russian], Vil'nyus (1979).
4. A. G. Kostyuk, "Statistical theory of plasticity of polycrystalline materials," Inzh. Zh., Mekh., No. 6 (1968).
5. T. N. Lin, "Physical theory of plasticity," The New in Foreign Science [in Russian], No. 7, Mir, Moscow (1976).
6. R. Hunnicomb, Plastic Deformation of Metals [Russian translation], Mir, Moscow (1972).
7. J. Eshelby, Continual Theory of Dislocations [Russian translation], IL, Moscow (1968).
8. A. G. Kostyuk, "Initial flow surface of a polycrystalline material," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 2 (1971).

PARAMETRIC RESONANCE IN A STRATIFIED FLUID

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Parametric resonance is one of the widespread types of instability of mechanical systems. A somewhat broader class of phenomena is called parametrically excited oscillations. The mathematical definition of this class of oscillations is ordinarily given [1] for systems whose equations of motion reduce to ordinary differential equations in the time. Parametric oscillations are related to the periodic dependence of the coefficients (parameters) of these equations on the time. Such oscillations are distinct from forced oscillations for which the explicit time dependence is contained only additively, in the form of periodic forces, in the equations. The Mathieu equation and its generalization are a standard example of parametric oscillation equations. The experimental work of Faraday [2], in which the oscillations of a free fluid surface in a vessel were studied, was the first investigation of parametric oscillations. However, mainly applications to solid and elastic bodies [1, 3, 4] were developed later. The exception is the problem of the oscillations of a free fluid surface in a vertically oscillating vessel. It has been shown [5-7] that in a linear approximation, the displacement of a free surface reduces to a Mathieu equation, and resonance frequencies therefore exist for which the surface turns out to be unstable. Taking account of the viscosity in this problem is presented in [8]. Only in the past decade have investigations been started on the parametric instabilities of more complicated flows. Parametric resonance in convection problems was studied in [9, 10]. The stability of Rossby waves was investigated in [11-14]. The papers [15, 16] are devoted to the instability of internal waves in a stratified fluid. A number of considerations on the possibility of the growth of fine-scale perturbations in the internal wave background is presented in [15].

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A theoretical investigation of the parametric instability of a plane internal wave in the Boussinesq approximation is contained in [16]. It is shown that a finite-amplitude wave can be unstable. In the small amplitude limit the parametric instability goes over into known [17] resonance interaction of waves.

In this paper the parametric resonance is studied in a stratified fluid. In the cases of a vertically oscillating vessel with a fluid and a horizontal plane-parallel flow, instability conditions are obtained. It is shown that an instability of the same kind holds in internal waves. Here the idea of the similarity of the physical conditions for the fluid motion in the oscillating vessel and in the wave is important. The distinctions are that oscillations in the wave are not solid-state and their frequencies are not given arbitrarily. If a long internal wave is considered, then locally (for shortwave perturbations) the conditions turn out to be close to a solid-state oscillating fluid. The type of internal wave instability studied is omitted from consideration in [16]. A study of the mechanisms by which destruction of the internal waves can occur is of great interest in connection with oceanology applications [15-18]. This latter reduces to the origination of zones in the wave, in which the density grows upward. Let us also note the interesting hypothesis [11, 12, 15, 16] which is that the parametric instability of wave motions can be a mechanism of the "loss of predictability" of the flow and the generation of turbulence.

1. Let us consider a rectangular vessel filled with an ideal incompressible fluid. At the initial instant ($t = 0$) the fluid occupies the volume $0 < x < a$, $0 < y < b$, $0 < z < c$. The fluid density is $\rho_0 = Ae^{-\beta y}$ ($A > 0$; β are constants). The homogeneous gravity field has only a y-component $(0, -g, 0)$. The buoyancy frequency (Brent-Wyaisyal) is $N^2 = \beta g = \text{const}$. The vessel moves in the y direction at the velocity $\dot{Y}(t) \equiv dY/dt$, where $Y(t)$ is a periodic function (finite-amplitude oscillations). The impenetrability conditions are satisfied on the vessel boundaries. The fluid state of rest relative to the vessel is the solution of the motion equations. Its stability must be investigated.

Let us go over a coordinate system coupled to the vessel

$$\bar{x} = x,\ \bar{y} = y - Y(t),\ \bar{z} = z,\ \bar{t} = t.$$

In these coordinates the equations of fluid motion have the same form as in the initial coordinates, except that the gravity field g has been replaced by $G \equiv g + \ddot{Y}$. The linearized system of perturbation equations has the form

$$\rho_0 u_t = -p_x,\ \rho_0 w_t = -p_z, \qquad (1.1)$$
$$\rho_0 v_t = -p_y - \rho G,\ \rho_t + \rho_0' v = 0,\quad u_x + v_y + w_z = 0,$$

where the bar above the x, y, z, t has been omitted. The x, y, z-components of the velocity perturbations are denoted by u, v, w and ρ, p are the density and pressure perturbations. The subscripts denote partial derivatives, and $\rho_0' \equiv d\rho_0/dy$. The substitution $\sigma = p/\rho_0$, $r = \rho/\rho_0$ reduces (1.1) to a system of equations with coefficients independent of x, y, z

$$u_t = -\sigma_x,\ w_t = -\sigma_z, \qquad (1.2)$$
$$v_t = -\sigma_y + \beta\sigma - Gr,\ r_t - \beta v = 0,\ u_x + v_y + w_z = 0,$$

that allows separation of variables. The equation

$$Dr_{tt} + \beta G(r_{xx} + r_{zz}) = 0, \qquad (1.3)$$

for r follows from (1.2), where $D = \Delta - \beta\partial/\partial y$ and Δ is the three-dimensional Laplace operator. The substitution $r = \varphi e^{\beta y/2}$ transfers (1.3) into

$$(\Delta - \beta^2/4)\varphi_{tt} + \beta G(\varphi_{xx} + \varphi_{zz}) = 0. \qquad (1.4)$$

The eigenfunction of the problem is

$$\varphi = R(t)k_1k_3 \cos k_1x \sin k_2y \cos k_3z, \qquad (1.5)$$

where $(k_1, k_2, k_3) \equiv \pi(n_1/a, n_2/b, n_3/c)$ and n_1, n_2, n_3 are arbitrary integers. The u, v, w components calculated from (1.2) and (1.5) satisfy the boundary conditions. There follows from (1.4), (1.5)

$$\ddot{R} + B(N^2 + \beta\ddot{Y})R = 0, \qquad (1.6)$$

where $B \equiv (k_1^2 + k_3^2)/(k^2 + \beta^2/4)$; $k^2 \equiv k_1^2 + k_2^2 + k_3^2$.

Because of the periodicity of Y(t), equation (1.6) is a Hill equation [19]. For Y = C cos ωt it reduces to the Mathieu equation

$$\ddot{R} + B(N^2 - \beta\omega^2 C \cos \omega t)R = 0,$$

whose canonical form is

$$R_{\tau\tau} + (a - 2q \cos 2\tau)R = 0, \tag{1.7}$$

where $\tau = \omega t/2$; $a = 4BN^2/\omega^2$; $q = 2BC$. The stability of the solutions of (1.7) has been studied in detail in [19, 20]. The unstable domains in the a, q plane (for $a > 0$) are "tongues" emerging from the points $a = m^2$, m = 1, 2, 3, ... For samll amplitudes C of the vessel oscillations the solutions of (1.7) are unstable in narrow zones around the points

$$\omega = 2NB^{1/2}/m. \tag{1.8}$$

Such an instability is called parametric resonance [1, 3, 4], and the number m is the order of the resonance. Since B < 1, for the order of resonance m to exist we must have $\omega < 2N/m$. For given vessel dimensions a, b, c and the buoyancy frequency N a four-parameter countable set (in n_1, n_2, n_3, m) of vessel oscillation frequencies ω (1.8) exists for which resonance holds. In all cases, the instability will be understood, here and below, as the exponential growth of the solutions as $t \to \infty$.

The preceding results have been obtained for equilibrium density stratification $N^2 \equiv \beta g > 0$. At the same time, it follows from (1.7) that upon compliance with the conditions

$$\sqrt{2B|N^2|}/\omega < CB|\beta| < 1/2 + 2B|N^2|/\omega^2 \tag{1.9}$$

the vessel oscillations make the state stable with the growth of the density upward ($N^2 < 0$). If the oscillation amplitude C is small, then the right side in (1.9) is always satisfied. The left side yields the condition

$$\omega C > (2g/|\beta|B)^{1/2}. \tag{1.10}$$

The stabilization property under discussion for the nonequilibrium state is an analog of the known result for a pendulum for which the upper position becomes stable during oscillations of the point of suspension [21]. Still closer analogs are the stabilization of the Rayleigh—Taylor [22] and convection [23] instabilities by oscillations. However, in contrast to these cases, stabilization of an ideal nonequilibrium stratified fluid has not been achieved successfully. Spoilage of the inequality as $B \to 0$ corresponds to this in (1.10). Small values of B are achieved either for $k_1^2 + k_3^2 \to 0$ or for $k_2 \to \infty$. Because of the limited size of the vessel, the first case be eliminated. Hence, short waves in the vertical direction are dangerous. Stabilization can possibly be achieved here by introducing viscosity.

Another interesting result of (1.7) is the flow instability in the absence of a gravity field g = 0. The instability condition (approximate), [20] is

$$|C| > (2|\beta|)^{-1}.$$

Such an instability can turn out to be important for predicting the behavior of a stratified fluid under weightless conditions.

2. Taking account of fluid viscosity results in replacement of the operator ∂/∂t in front of the velocity components in (1.1) by ∂/∂t − νΔ. Under the condition of constancy of the kinematic viscosity coefficient ν = const, we obtain the following equation in place of (1.4):

$$(\Delta - \beta^2/4)[\varphi_{tt} - \nu(\Delta + \beta^2/4 + \beta\partial/\partial y)\varphi_t] + \beta G(\varphi_{xx} + \varphi_{zz}) = 0.$$

The solution of the problem with the adhesion conditions on the vessel boundaries satisfied is quite complex and can be considered as a generalization [8, 10]. Let us consider an infinite vessel. After separation of variables

$$\varphi = R(t)\, e^{i(k_1 x + k_2 y + k_3 z)}$$

we obtain the equation

$$\ddot{R} + \lambda\dot{R} + B(N^2 + \beta\ddot{Y})R = 0, \tag{2.1}$$

which is a generalization of (1.6). This equation is a Hill equation (or a Mathieu equation for Y = C cos ωt) with friction. The form of the friction coefficient is unusual:

$$\lambda = \nu(k^2 - \beta^2/4 - ik_2\beta).$$

For instance, both damping and growth can be obtained from (2.1) for $N^2 + \beta\ddot{Y} = 0$ for different k. Such a behavior of R(t) is related to the unboundedness of the selected solutions of (1.1) for any fixed t. In the Boussinesq approximation (see below), the solutions are bounded, and $\lambda = \nu k^2$, which always corresponds to damping.

3. A direct extension of the problems considered is the problem of the vertical oscillations of a horizontal plane-parallel ideal incompressible stratified fluid stream. The stream is along the x axis, and the velocity is $U = U(y)$. Retaining the notation from Sec. 1, we obtain a system of equations in the linear perturbations:

$$\rho_0(Lu + U'v) = -p_x,\ \rho_0 Lv = -p_y - \rho G,\ \rho_0 Lw = -p_z, \qquad (3.1)$$
$$L\rho + \rho_0' v = 0, \quad u_x + v_y + w_z = 0,$$

where $L \equiv \partial/\partial t + U\partial/\partial x$. From (3.1) there follows $(\Delta + \beta\partial/\partial y)L^2\rho + \beta G(\rho_{xx} + \rho_{zz}) - 2(U'L\rho_x)_y = 0$. Investigation of the stability of the solutions of this equation is extremely complicated. However, and equation of the type (1.3) is again obtained for perturbations independent of the coordinate x:

$$(\partial^2/\partial z^2 + \partial^2/\partial y^2 + \beta\partial/\partial y)\rho_{tt} + \beta G\rho_{zz} = 0,$$

which is solved by the substitution $\rho = \varphi e^{-\beta y/2}$ with subsequent separation of variables. The results obtained are the same as in Secs. 1 and 2, with the sole difference that $k_1 = 0$.

4. It was shown in the previous examples that the oscillations of a volume of stratified fluid as a whole can result in instability by the parametric resonance mechanism. It is natural to expect that both non-solid-state (differential) oscillations of a medium, and particularly internal waves (see the Introduction) possess similar properties. The main difficulties in investigating the stability of internal motions in a stratified fluid are the complexity of the original equations, and the absence of sufficiently simple particular solutions. The outcome is to go over to approximations in either the solutions or in the equations directly. An approximation in the solution is understood to be that the fundamental wave motion being investigated for stability is given approximately (for instance, in the form of a finite number of terms in the amplitude series). Mathematically, this operation has the meaning of replacing the coefficients in the equations being investigated for stability by their approximate (analytically more simple) values. Examples of direct simplification of the equations of motion are the Boussinesq approximation [17], or the β-plane approximation [11, 13, 24]. The question of the mathematical correctness of such a consideration is extremely complex. At the same time, the scientific and practical values of the problem is a justification of activity at the "physical level of strictness." Two examples of approximations in the solutions and in the equations for the problem of internal wave stability are elucidated below.

An ideal incompressible stratified fluid filling all of space is given. The homogeneous gravity field $-g$ is directed along the y axis. The unperturbed fluid density $\rho_0 = Ae^{-\beta y}$, the buoyancy frequency is $N^2 = \beta g = \text{const}$ (the notation is the same as in Sec. 1). Let us examine the problem of stability of an internal wave of a particular form, where we limit ourselves to linear expressions in the amplitude in giving it. The form of such a wave is given by the representation

$$u = 0,\ v = Y_t(x, t) = \partial Y/\partial t,$$

where u and v are the x and y components of the velocity. The function Y(x, t) is a traveling or standing wave of the form $\cos(kx - Nt)$ or $\cos kx \cos Nt$, etc. The frequency of the wave agrees with the buoyancy frequency N. By analogy with Sec. 1, we perform the coordinate transformation

$$\bar{x} = x,\ \bar{y} = y - Y(x, t),\ \bar{z} = z,\ \bar{t} = t,$$

which corresponds to going over to coordinates "oscillating" together with the fluid. Thus, the density in the internal wave is independent of the time $\rho = \rho_0(\bar{y})$. We introduce $\bar{v} = v - Y_t$ for the vertical component v so that the ground state is rest $u = 0$, $\bar{v} = 0$. The linearized system of equations, the analog of (1.1), has the form

$$\rho_0 u_t = -p_x + Y_x p_y,\ \rho_0 w_t = -p_z. \qquad (4.1)$$

$$\rho_0(v_t + Y_{xt}u) = -p_y - \rho(g + Y_{tt}),$$
$$\rho_t + \rho_0'(v - Y_x u) = 0, \quad u_x - Y_x u_y + v_y + w_z = 0.$$

The bar is omitted above the notation. Let us examine a perturbation of particular form:

$$v = v(x, t), \; r = r(x, t), \; u = w = p = 0.$$

For $r = \rho/\rho_0$

$$r_{tt} + (N^2 + Y_{tt})r = 0. \tag{4.2}$$

This equation is the analog of (1.6). For $Y = \Phi(x)\cos Nt$, Eq. (4.2) reduces to the Mathieu equation in which x plays the part of a parameter. Since the forcing frequency equals the fundamental, a second resonance occurs [3, 20], and the wave is unstable even for small amplitudes Φ. For the traveling wave $Y = C\cos(kx - Nt)$ and (4.2) also reduces to a Mathieu equation but only at the points $\sin kx = 0$. Instability hence also follows in this case. Let us yet note that terms with Y enter the system (4.1) differently. The derivative is $Y_x \sim C/\lambda$, where C is the amplitude and λ the wavelength. If this ratio is considered small and is neglected, then (4.1) reduces to (1.1) with the sole difference that the quantity Y_{tt} depends on the coordinate x. For shortwave perturbations in x this system of equations agrees with (1.1) in a first approximation.

Now, let us consider the approach of approximations in the equations. The Boussinesq approximation [17] is a known simplification of the equations of motion of a stratified fluid

$$d\mathbf{u}/dt = -\nabla p/\bar{\rho} - \theta\mathbf{g}, \; d\theta/dt - \beta v = 0, \; \operatorname{div}\mathbf{u} = 0, \tag{4.3}$$

where $\mathbf{u}$ is the velocity vector, $\theta \equiv (\rho - \bar{\rho})/\bar{\rho}$; ρ is the total density which differs little from $\bar{\rho}$ = const; $\beta = \beta(y) \equiv -\rho_0'(y)/\bar{\rho}$. For $\beta > 0$ the system (4.3) has exact solutions, plane waves:

$$(u, v, \theta, p) = (-l/k, 1, i\beta/\omega, -\omega l/k^2)Ce^{i\psi}, \tag{4.4}$$

where $\omega = \pm kN/\sqrt{k^2 + l^2}$; C = const; $\psi = kx + ly + \omega t$. It is shown in [16] that these solutions can be unstable. At the same time, it turns out that part of terms of the equations of motion that were discarded in obtaining (4.3) can also yield a parametric instability. To prove this assertion it is sufficient to note that the use of (4.3) in the problem of the oscillating vessel (see Sec. 1) does not generally result in instability. It is clear that such an approach is incorrect: Small discarded terms of the equations yield an effect which cumulates in a resonance manner. To maintain the instability discussed in Sec. 1 in the equations, it is necessary to return the term $\theta\, du/dt$ discarded earlier, into (4.3). This operation is equivalent to goving over to the Boussinesq approximation in the coordinate system coupled to the vessel. Afterwards, the linearized system of equations of the stability problem follows from (4.3):

$$v_t = -p_y/\bar{\rho} - \theta(g + \ddot{Y}), \quad u_t = -p_x/\bar{\rho}, \quad w_t = -p_z/\bar{\rho}, \quad \theta_t - \beta v = 0,$$
$$u_x + v_y + w_z = 0. \tag{4.4*}$$

A comparison with the system (1.1) shows that (4.4) is a simplified variation. It follows from (4.4) that

$$\Delta\alpha_{tt} + \beta G(\alpha_{xx} + \alpha_{zz}) = 0, \tag{4.5}$$

where $\alpha \equiv \theta/\beta$; $G \equiv g + \ddot{Y}$. Furthermore, the problem is solved by separation of variables (see Sec. 1). For β = const Eq. (4.5) is a simplification of (1.4). Taking account of viscosity results in the appearance of a dissipation term in the most simple form in (4.5)

$$\Delta(\alpha_{tt} - \nu\Delta\alpha_t) + \beta G(\alpha_{xx} + \alpha_{zz}) = 0.$$

Exactly the same, taking account of the above-mentioned term of the equations results in the appearance of an additional (compared with [16]) instability in the solution (4.4). Let us write the equation of motion in a coordinate system with x axis along the wave vector **k**, and the y axis along the velocity vector in the wave. The velocity and density fields (4.4) take the form

$$(U, V, \Theta) = (0, 1, i\beta/\omega)Ce^{i(\varkappa x - \omega t)},$$

*There are two Eqs. (4.4) in the Russian original — Publisher.

where $\omega = N\cos\varphi$; φ is the angle between the x axis and the horizontal plane, and $\varkappa^2 \equiv k^2 + l^2$. The linearized system of equations of the stability problem has the form

$$Lu = -p_x - g\theta\sin\varphi,\ Lv + V_x u = -p_y - \theta(g\cos\varphi + V_t),$$
$$Lw = -p_z,\ u_x + v_y + w_z = 0,\ L\theta + \Theta_x u - \beta v = 0, \tag{4.6}$$

where $L \equiv \partial/\partial t + V\partial/\partial y$. Let us consider perturbations of a particular form

$$v = v(x,\ t),\ \theta = \theta(x,\ t),\ p = p(x,\ t),\ u = w = 0.$$

It follows from (4.6)

$$\theta_{tt} + (N^2\cos^2\varphi + \beta V_t\cos\varphi)\theta = 0.$$

Since the frequency of variation V equals $\omega = N\cos\varphi$, second resonance occurs for $V = C\cos(\varkappa x - \omega t)$. Let us emphasize that the results of this section are illustrative in nature and not proofs:

In conclusion, we note the following. The problem of investigating the stability in a linear approximation of the equilibrium of an ideal stratified fluid in a vertically oscillating vessel reduces to solving the Mathieu equation (1.7). For an equilibrium density stratification, an instability of parametric resonance type is possible. Stabilization by oscillations of the nonequilibrium stratification is possible only for part of the spectrum.

Perturbations of a particular kind exist in a horizontal plane-parallel stream of stratified fluid subjected to vertical oscillations, for which the same results are valid as for the oscillating vessel.

An approximate investigation of the stability of internal waves in an infinite stratified fluid shows that an instability of the parametric resonance type holds that is similar in type to the instability in an oscillating vessel.

Let us formulate the following as a development of the general representation about instability mechanisms in a stratified fluid. If the fluid oscillations are such that the mass flow rate has a component normal to the constant density surfaces, then exponential growth of the linear perturbations is possible by the mechanism of parametric resonance.

LITERATURE CITED

1. G. Schmidt, Parametric Oscillations [Russian translation], Mir, Moscow (1978).
2. M. Faraday, "On a peculiar class of acoustical figures, and on certain forms assumed by a group of particles upon vibrating elastic surfaces," Philos. Trans. R. Soc., 121, 299 (1831).
3. L. D. Landau and E. M. Lifshitz, Mechanics, Pergamon (1975).
4. V. A. Yakubovich and V. M. Starzhinskii, Linear Differential Equations with Periodic Coefficients and Their Application [in Russian], Nauka, Moscow (1972).
5. N. N. Moiseev and V. V. Rumyantsev, Dynamics of a Body with Cavities Containing Liquids [in Russian], Nauka, Moscow (1965).
6. L. N. Sretenskii, Theory of Wave Motions of Fluids [in Russian], Nauka, Moscow (1977).
7. T. B. Benjamin and F. Ursell, "The stability of the plane free surface of a liquid in vertical motion," Proc. R. Soc., 225, 505 (1954).
8. S. I. Krushinskaya, "Oscillations of a heavy viscous fluid in a moving vessel," Zh. Vychisl. Mat. Mat. Fiz., 5, No. 3 (1965).
9. G. Z. Gershuni and E. M. Zhukovitskii, Convective Stability of an Incompressible Fluid [in Russian], Nauka, Moscow (1972).
10. G. S. Markman and A. L. Urintsev, "On the parametric excitation of convective motion in a fluid heated from above," Izv. Sev. Kavkaz. Nauch. Tsentr. Vyssh. Shkol. Estestv. Nauka, No. 1 (1977).
11. E. N. Lorenz, "Barotropic instability of Rossby wave motion," J. Atmos. Sci., 29, 258 (1972).
12. D. K. Lilly, "A note on barotropic instability and predictability," J. Atmos. Sci., 30, No. 1 (1973).
13. R. P. Mied, "The instabilities of finite-amplitude barotropic Rossby waves," J. Fluid Mech., 86, No. 2 (1978).

14. F. V. Dolzhanskii, M. V. Kurganskii, and Yu. L. Chernous'sko, "Laboratory and theoretical investigation of barotropic Rossby waves in a rotating annular channel," Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana, 15, No. 6 (1979).
15. A. D. McEwan and R. M. Robinson, "Parametric instability of internal gravity waves," J. Fluid Mech., 67, No. 4 (1975).
16. R. P. Mied, "The occurrence of parametric instability of internal gravity waves," J. Fluid Mech., 78, No. 4 (1976).
17. O. M. Phillips, Dynamics of the Upper Ocean, Cambridge Univ. Press (1980).
18. I. Orlanski, "On the breaking of standing internal gravity waves," J. Fluid Mech., 54, No. 3 (1972).
19. E. T. Whittaker and G. N. Watson, Modern Analysis [in Russian], Vol. 1, FML, Moscow (1963).
20. M. Abramowitz and I. Stegan, Handbook on Special Functions [Russian translation], Nauka, Moscow (1979).
21. P. L. Kapitsa, "Dynamic stability of a pendulum with an oscillation point of suspension," Zh. Eksp. Teor. Fiz., 21, No. 5 (1951).
22. V. A. Briksman, "Parametric stabilization of the fluid interface," Dokl. Akad. Nauk SSSR, 226, No. 5 (1976).
23. S. M. Zen'kovskaya and I. B. Simonenko, "On the influence of high-frequency vibrations on the origin of convection," Izv. Akad. Nauk SSSR, Mekhan. Zhidk. Gaza, No. 5 (1966).
24. G. Batchelor, Introduction to the Dynamics of Fluids [Russian translation], Mir, Moscow (1973).